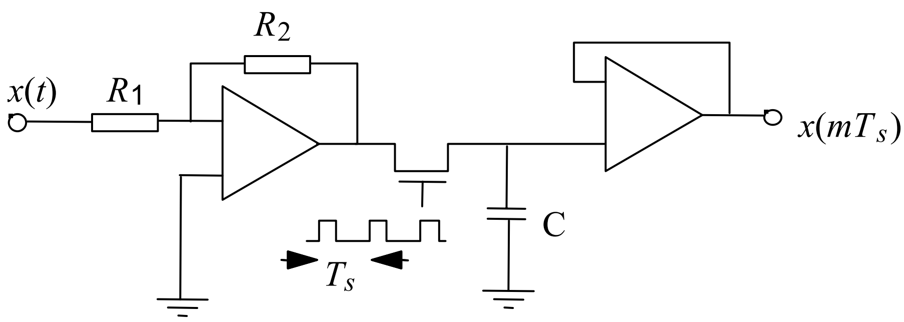


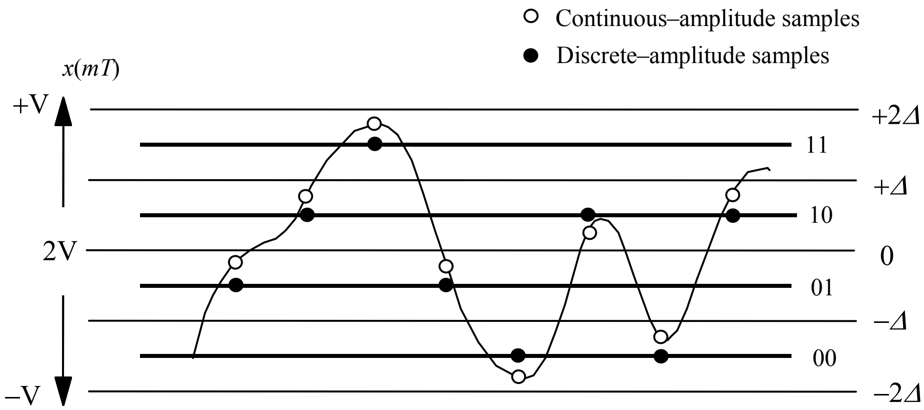
Figure 1.20 A simplified sample-and-hold circuit diagram.

signal, then the repetitions of the signal spectra are separated as shown in Figure 1.19. In this case, the analog signal can be recovered by passing the sampled signal through an analog low-pass filter with a cut-off frequency of F_s. If the sampling frequency is less than $2F_s$, then the adjacent repetitions of the spectrum overlap and the original spectrum cannot be recovered. The distortion, due to an insufficiently high sampling rate, is irrevocable and is known as *aliasing*. This observation is the basis of the *Nyquist sampling theorem* which states: a band-limited continuous-time signal, with a highest frequency content (bandwidth) of B Hz, can be recovered from its samples provided that the sampling speed $F_s>2B$ samples per second.

In practice sampling is achieved using an electronic switch that allows a capacitor to charge up or down to the level of the input voltage once every T_s seconds as illustrated in Figure 1.20. The sample-and-hold signal can be modelled as the output of a filter with a rectangular impulse response, and with the impulse–train–sampled signal as the input as illustrated in Figure1.19.

1.4.2 Quantisation

For digital signal processing, continuous-amplitude samples from the sample-and-hold are quantised and mapped into n-bit binary digits. For quantisation to n bits, the amplitude range of the signal is divided into 2^n discrete levels, and each sample is quantised to the nearest quantisation



level, and then mapped to the binary code assigned to that level. Figure 1.21 illustrates the quantisation of a signal into 4 discrete levels. Quantisation is a many-to-one mapping, in that all the values that fall within the continuum of a quantisation band are mapped to the centre of the band. The mapping between an analog sample $x_a(m)$ and its quantised value $x(m)$ can be expressed as

$$x(m)=Q[x_a(m)] \tag{1.25}$$

where $Q[\cdot]$ is the quantising function.

The performance of a quantiser is measured by signal–to–quantisation noise ratio SQNR per bit. The quantisation noise is defined as

$$e(m)=x(m)-x_a(m) \tag{1.26}$$

Now consider an n-bit quantiser with an amplitude range of $\pm V$ volts. The quantisation step size is $\Delta=2V/2^n$. Assuming that the quantisation noise is a zero-mean uniform process with an amplitude range of $\pm\Delta/2$ we can express the noise power as

$$\begin{aligned}\mathcal{E}\left[e^2(m)\right]&=\int_{-\Delta/2}^{\Delta/2} f_E(e(m))\,e^2(m)\,de(m)=\frac{1}{\Delta}\int_{-\Delta/2}^{\Delta/2} e^2(m)\,de(m)\\ &=\frac{\Delta^2}{12}=\frac{V^2 2^{-2n}}{3}\end{aligned} \tag{1.27}$$

where $f_E(e(m)) = 1/\Delta$ is the uniform probability density function of the noise. Using Equation (1.27) he signal–to–quantisation noise ratio is given by

$$\begin{aligned} SQNR(n) &= 10\log_{10}\left(\frac{\mathcal{E}[x^2(m)]}{\mathcal{E}[e^2(m)]}\right) = 10\log_{10}\left(\frac{P_{\text{Signal}}}{V^2 2^{-2n}/3}\right) \\ &= 10\log_{10}3 - 10\log_{10}\left(\frac{V^2}{P_{\text{Signal}}}\right) + 10\log_{10}2^{2n} \qquad (1.28) \\ &= 4.77 - \alpha + 6n \end{aligned}$$

where P_{signal} is the mean signal power, and α is the ratio in decibels of the peak signal power V^2 to the mean signal power P_{signal}. Therefore, from Equation (1.28) every additional bit in an analog to digital converter results in 6 dB improvement in signal–to–quantisation noise ratio.